Solutions to Seminar 3 ECON4930 Tuesday 8 March

1a)

$$\frac{\partial L}{\partial e_{jt}^{H}} = p_t(\sum_{j=i}^{N} e_{jt}^{H} + \sum_{i=1}^{M} e_{it}^{Th}) - \lambda_{jt} \le 0 (= 0 \text{ for } e_{jt}^{H} > 0)$$
$$\frac{\partial L}{\partial R_{jt}} = -\lambda_{jt} + \lambda_{j,t+1} - \gamma_{jt} \le 0 (= 0 \text{ for } R_{jt} > 0)$$
$$\frac{\partial L}{\partial e_{it}^{Th}} = p_t(\sum_{j=i}^{N} e_{jt}^{H} + \sum_{i=1}^{M} e_{it}^{Th}) - c_i'(e_{it}^{Th}) - \theta_{it} \le 0 \ (= 0 \text{ for } e_{it}^{Th} > 0)$$
$$\lambda_{jt} \ge 0 (= 0 \text{ for } R_{jt} < R_{j,t-1} + w_{jt} - e_{jt}^{H})$$
$$\gamma_{jt} \ge 0 (= 0 \text{ for } R_{jt} < \overline{R}_j)$$
$$\theta_{it} \ge 0 (= 0 \text{ for } e_{it}^{Th} < \overline{e}_i^{Th}) , \quad i = 1,..,M , t = 1,..,T , j = 1,..,N$$

1b) Assume that T is one year, or that T can be divided into integer years and then look at one year.

A base-load hydropower plant: $e_{jt}^{H} > 0$ for t covering i) all T periods, ii) e.g.75 % of the periods.

A base-load thermal plant: a) $e_{it}^{Th} = \overline{e}_i^{Th}$ for all T periods, b) e.g. 75 % of the periods, $e_{it}^{Th} \ge 0$ for the other periods.

A peak-load hydro plant: $e_{jt}^{H} \le \overline{e}_{jt}^{H}$ for some t, (20-25%), $e_{jt}^{H} = 0$ for the other time periods

A peak-load thermal plant: $e_{it}^{Th} = \overline{e}_i^{Th}$ for some t, (20-25%), $e_{it}^{Th} = 0$ for the other time periods

1c) We must have all plants (no plants continuing accumulating water) being at the reservoir capacity when the price increases for the next period, all plants being emptied when there is a price increase for the period, all reservoirs being emptied when there is a price decrease, and we must have a unique merit order ranking of all thermal plants.

2a) There is an installed maximal power capacity; the energy production depends on the wind, sun, river flow during the period t. We need a conversion from capacity to produce energy to actual production. First of all a windmill capacity is usually expressed in a power unit, e.g. MW. We then assume that capacity is used at the same rate thoughout our period so we can express output in MWh. We introduce a wind factor α_t based on the strenght of the wind that we can normalise such that the wind factor is 1 for the maximal strenght of wind (25 m/s) before the windmill is turned off (rotors parked in a no-production position): $e_t^I = \alpha_t \overline{e}^I , \alpha_t \ge 0$. We then have $e_t^I = 0 \text{ for } \alpha_t \le \alpha^{Min}, e_t^I = 0 \text{ for } \alpha_t \ge 1$

For a run-of-the-river plant α_t may be interpreted as the river flow, then α^{Min} is zero, and

α_t =1 is interpreted as full capacity utilisation, this flow factro can then be greater than 1, but the production is constant equal to the maximal.

2b)

$$\frac{\partial L}{\partial e_t^H} = p_t(e_t^H + e_t^C + e_t^N + e_t^I) - \lambda_t \le 0 \ (= 0 \text{ for } e_t^H > 0)$$
$$\frac{\partial L}{\partial e_t^C} = p_t(e_t^H + e_t^C + e_t^N + e_t^I) - c_C'(e_t^C) - \theta_t^C \le 0 \ (= 0 \text{ for } e_t^C > 0)$$
$$\frac{\partial L}{\partial e_t^C} = p_t(e_t^H + e_t^C + e_t^N + e_t^I) - c_N'(e_t^N) - \theta_t^N \le 0 \ (= 0 \text{ for } e_t^N > 0)$$
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \ (= 0 \text{ for } R_t > 0)$$
$$\lambda_t \ge 0 \ (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H)$$
$$\gamma_t \ge 0 \ (= 0 \text{ for } R_t < \bar{R})$$
$$\theta_t^C \ge 0 \ (= 0 \text{ for } e_t^C < \bar{e}^C)$$
$$\theta_t^N \ge 0 \ (= 0 \text{ for } e_t^N < \bar{e}^N), \quad t = 1,..,T$$

2c)

$$\frac{\partial(Obj.f.)}{\partial \bar{R}} = \frac{\partial L}{\partial \bar{R}} = \sum_{t=1}^{T} \gamma_t$$ NB! The shadow price will be zero for a number of periods

$$\frac{\partial(Obj.f.)}{\partial \bar{e}^I} = \frac{\partial L}{\partial \bar{e}^I} = \sum_{t=1}^{T} \alpha_t p_t$$ NB! Production will be zero for the periods where the wind factor is less than the minimum, and for periods where the wind factor is greater than 1. So these periods must be removed from the sum above. Interpretation for a run-of-the-river plant followes from answers to 2a).

2d) See the figures in slides for Lecture 5. Manipulate the demand function for period 2 to get an intersection of demand curves within the reservoir limits.

2e) Pumped storage economics; e_t^{PH} is the energy in kWh pumped up during period t, $e_{t+1}^{H_{PH}}$ is the amount of energy we get releasing the pumped water onto the turbines again in the next:

$$e_t^{PH} = e_{t+1}^{H_{PH}} = \beta e_{t+1}^{PH}, \beta < 1$$
$$p_t e_t^{PH} \le p_{t+1} e_{t+1}^{H_{PH}} \Rightarrow p_t \le p_{t+1} \frac{\beta e_{t+1}^{PH}}{e_t^{PH}} = \beta p_{t+1}$$

(β: 0.70-0.85)

Pumped storage increases the inflow in period t+1.

Using a discrete time period we must make an assumption that we pump in continuous time within the period. An upper limit on the pumping capacity per unit of time period may be considered.